The muon g-2 in a $SU(7)$ left-right symmetric model with mirror fermions

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Abstract

We have studied a left-right symmetric model with mirror fermions based in a grand unified SU(7) model in order to account for the muon anomaly. The Higgs sector of the model contains two Higgs doublets and the hierarchy condition $\upsilon_L \ll \upsilon_R$ can be achieved by using two additional Higgs singlets, one even and other odd under $\mathcal{D}$-parity. We show that there is a wide range of values for the mass parameters of the model that is consistent with the $g-2$ lepton anomalies. Radiative correction to the mass of the ordinary fermions are shown to be small.

1 Introduction

Some years ago it was recognized that the measurements of $a_l = \frac{g_l-2}{2}$ for leptons (commonly referred to as the lepton anomaly) can be an interesting window to discover New Physic beyond the Standard Model.

The recent theoretical results for the electron anomaly are now known at order α^5 [1] in its QED contribution and at two-loops for the electroweak corrections [2] for the muon anomaly. The present values are: $\Delta a_e \simeq (1.24 \pm 0.95) \times 10^{-11}$, for the electron and $\Delta a_\mu \simeq (2 \pm 2) \times 10^{-9}$, for the muon. In fact, the results reported by the Muon $(g-2)$ Collaboration [3] combined with the most recent theoretical calculations have shown that there remains a discrepancy with the SM theoretical calculations at a confidence level from 0.7σ to 3.2 σ, according to the values chosen for the hadronic contributions. Using $e+e-$ data, the SM prediction for the muon $g_\mu - 2$ deviates from the present experimental value [4][5][6][7][8] by $2\sigma - 3\sigma$, if the hadronic light-by-light contribution is used instead of the hadronic τ decay data . Among all contributions yielding corrections to the muon anomaly, the hadronic contributions are less accurate due to the hadronic vacuum polarization effects in the diagrams which use data

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inputs coming from the e^+e^- annihilation cross section and the hadronic $\tau-$ decay. It is not clear at present whether the value from $\tau-$ decay data can be improved much further due to the difficulty in evaluating more precisely the effect of isospin breaking.

In fact, these measurements have provided the highest accuracy for the predictions of theories for strong, weak and electromagnetic interactions because they have reached a fabulous relative precision of 0.5 parts per million (ppm) in the determination of a_μ. However, if this discrepancy for the muon anomaly remains, it is possible that we are under a window open for New Physics at a high energy scale, Λ. The study of the muon anomaly becomes relevant because it is more sensitive to interactions that are not predicted in the SM but that can be reached at the CERN large hadron collider (LHC) with $\sqrt{s} = 14\,TeV$.

On the theoretical side, if we take into account the effects of virtual massive particles in the diagrams contributing to the lepton anomaly, the corrections to the anomalies are expected to be of the order $\left(\frac{m_\mu}{m_e}\right)^2 \sim 4 \times 10^4$ for the muon, and of the order $\left(\frac{m_\tau}{m_e}\right)^2 \sim 1.2 \times 10^7$ for the tau. The same huge enhancement factor would also affect the contributions coming from degrees of freedom beyond the SM, so that the measurement of the $\tau-$ anomaly would represent the best opportunity to detect new physics. Unfortunately, the very short lifetime of the τ- lepton which, precisely because of its high mass, can also decay into hadronic states, makes such a measurement impossible at present. This is the reason for the emphasis on the muon anomaly.

Many models beyond the Standard Model have been proposed in order to explain the discrepancy of the muon $g_\mu - 2$. This is the case for example in E_6 GUT models [9][10] , supersymmetry [11] and left - right (L-R) models with mirror fermions [12]. For a review of models for New Physic see Ref.[13].

In this paper we have studied a left-right symmetric model with mirror fermions in the $SU(7)$ context using a minimal set of Higgs fields that consists in two doublets and two singlets. One essential ingredient in our model is the incorporation of $\mathcal{D}-$parity to induce the breaking of $SU(2)_R$ through the vacuum expectation value (v.e.v.) of an odd Higgs relative to $\mathcal{D}-$parity [14]. We expect that the breaking scale of $SU(2)_R$ will be not very far from the electroweak scale, let us say $\sim TeV$ due to introduction of an even $\mathcal{D}-$parity Higgs singlet which gain a v.e.v. in the GUT scale before the $\mathcal{D}-$parity odd Higgs.

Our paper is organized as follow. In Section 2 we introduce our left-right symmetrical model with mirror fermions based in a $SU(7)$ GUT. In Section 3 we study the Higgs sector and its interactions with charged leptons that is relevant for the lepton anomaly and its radiative mass. In Section 4 we analyze the constraints on the relevant parameters of the model. In Section 5 we present our conclusions.

2 A left-right symmetric model with mirror fermions in SU(7)

Left-right symmetric models are expected to be a natural consequence of $SU(3)_C\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_Y$. This extension of the standard model is also expected to be a sub-group of some grand unified model. The fermionic content of the fundamental representations can vary, but an economical choice consists of mirror fermions, related by a parity symmetry. Mirror fermions are suggested for example in $SO(2n)$, $SO(2n+1)$ [15], $SU(n)$ [16] ($n>5$) and E_8 [17] unifications models, as well as direct product group such as $SU(5)\otimes SU(5)$ [18].

In the L-R model with mirror fermions the particle content is described in Table 1 for the two first families of fermions.

Table 1

Ordinary fermions	Mirror fermions
$l_L=\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L,\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L\sim(\mathbf{1},\mathbf{2},\mathbf{1},-1)$	$L_R=\begin{pmatrix}N_E\\ E\end{pmatrix}_R,\begin{pmatrix}N_M\\ M\end{pmatrix}_R\sim(\mathbf{1},\mathbf{1},\mathbf{2},-1)$
$e_R,\mu_R\sim(\mathbf{1},\mathbf{1},\mathbf{1},-2)$	$E_L,\ M_L\sim(\mathbf{1},\mathbf{1},\mathbf{1},-2)$
$\nu_{eR},\nu_{\mu R}\sim(\mathbf{1},\mathbf{1},\mathbf{1},0)$	$N_{EL},N_{ML}\sim(\mathbf{1},\mathbf{1},\mathbf{1},0)$
$\begin{pmatrix}u\\ d\end{pmatrix}_L,\begin{pmatrix}c\\ s\end{pmatrix}_L\sim(\mathbf{3},\mathbf{2},\mathbf{1},1/3)$	$\begin{pmatrix}U\\ D\end{pmatrix}_R,\begin{pmatrix}C\\ S\end{pmatrix}_R\sim(\mathbf{3},\mathbf{1},\mathbf{2},1/3)$
$u_R,c_R\sim(\mathbf{3},\mathbf{1},\mathbf{1},4/3)$	$U_L,C_L\sim(\mathbf{3},\mathbf{1},\mathbf{1},4/3)$
$d_R,s_R\sim(\mathbf{3},\mathbf{1},\mathbf{1},-2/3)$	$D_L,S_L\sim(\mathbf{3},\mathbf{1},\mathbf{1},-2/3)$

Content of the first two families of ordinary fermions with its mirror partners and quantum numbers under $SU(3)_C\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_Y$.

In order to justify our choice of $SU(7)$ as the unification group, some points should be observed. First, as $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ is a maximal sub-group of $SU(5)$, then $SU(3)_C\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_Y\subset SU(5)\otimes SU(2)_R\subset SU(7)$. In fact [19] $SU(5)\otimes SU(2)\otimes U(1)_X$ is a maximal sub-group of $SU(7)$ and we can assume $SU(2)$ to have the right chirality $SU(2)_R$.

A second point is that the mass terms of leptons $\overline{l_{eL}}\chi_L e_R$ require Higgs representations $\chi_L\sim(\mathbf{1},\mathbf{2},\mathbf{1},1)$. Similarly the mass terms of the mirror partners $\overline{L_{ER}}\chi_R E_L$, require $\chi_R\sim(\mathbf{1},\mathbf{1},\mathbf{2},1)$. Mixing terms of the type $\overline{e_R}S_D E_L$, $\overline{\nu_R}S_D N_{EL}$ need $S_D\sim(\mathbf{1},\mathbf{1},\mathbf{1},0)$. Mass terms of the Majorana type $\overline{l_{eL}}\widetilde{\chi_L}N^C_{EL}$ need $\widetilde{\chi_L}\sim(\mathbf{1},\mathbf{2},\mathbf{1},-1)$ and $\overline{L_{ER}}\widetilde{\chi_R}\nu^C_{eR}$ need $\widetilde{\chi_R}\sim(\mathbf{1},\mathbf{1},\mathbf{2},-1)$ in order to give mass to neutrinos. The $\overline{N^C_{EL}}S_M N_{EL}$ and $\overline{\nu^C_{eR}}S_M\nu_{eR}$ terms are possible with $S_M\sim(\mathbf{1},\mathbf{1},\mathbf{1},0)$. Now, let us search for the representations of $\chi_{L,R}$, S_D and S_M in the $SU(7)$ context [20]. The multiplet fermions [21] are into the anomaly free combination[1] $\{\mathbf{1}\}\oplus\{\mathbf{7}\}\oplus\{\mathbf{21}^*\}\oplus\{\mathbf{35}\}$ corresponding to the spinor representation **64** of $SO(14)$ into which $SU(7)$ is embedded. In the previous multiplets, $\{\mathbf{21}\}$ is a 2-fold, $\{\mathbf{35}\}$ is a 4-fold and $\{\mathbf{7}\}$ is 6-fold of totally antisymmetric tensors.

[1] We are using {} for the $SU(7)$ components.

Let us note that **64** can contain two families of ordinary fermions with its respective mirror partners, for example the electron and muon families as it is showed in Table 1. The other families can be incorporated into other **64** spinorial representation. The branching rules for each component of the spinorial representation under $SU(5)\otimes SU(2)_R$, are [22]:

$$\begin{aligned}\{\mathbf{35}\} &= [\mathbf{10}^*,\mathbf{1}]\oplus[\mathbf{10},\mathbf{2}]\oplus[\mathbf{5},\mathbf{1}],\\ \{\mathbf{21}\} &= [\mathbf{10},\mathbf{1}]\oplus[\mathbf{5},\mathbf{2}]\oplus[\mathbf{1},\mathbf{1}],\\ \{\mathbf{7}\} &= [\mathbf{5},\mathbf{1}]\oplus[\mathbf{1},\mathbf{2}],\end{aligned} \tag{1}$$

and under $SU(3)_C\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_Y$

$$\begin{aligned}\{\mathbf{35}\} =& \underbrace{(\mathbf{1},\mathbf{1},\mathbf{1},-2)}_{e_R}\oplus\underbrace{(\mathbf{3},\mathbf{1},\mathbf{1},4/3)}_{u_R}\oplus\underbrace{(\mathbf{3},\mathbf{2},\mathbf{1},1/3)}_{\begin{pmatrix}c\\ s\end{pmatrix}_L}\oplus\underbrace{(\mathbf{1},\mathbf{1},\mathbf{1},-2)}_{E_L}\oplus\\ &\underbrace{(\mathbf{1},\mathbf{1},\mathbf{1},-2)}_{M_L}\oplus\underbrace{(\mathbf{3},\mathbf{1},\mathbf{1},4/3)}_{U_L}\oplus\underbrace{(\mathbf{3},\mathbf{1},\mathbf{1},4/3)}_{C_L}\oplus\underbrace{\mathbf{3},\mathbf{1},\mathbf{1},-2/3)}_{s_R}\oplus\\ &\underbrace{(\mathbf{1},\mathbf{2},\mathbf{1},-1)}_{\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L}\oplus\underbrace{(\mathbf{3},\mathbf{1},\mathbf{2},1/3)}_{\begin{pmatrix}U\\ D\end{pmatrix}_R}\oplus\underbrace{(\mathbf{3},\mathbf{1},\mathbf{2},1/3)}_{\begin{pmatrix}C\\ S\end{pmatrix}_R},\end{aligned} \tag{2}$$

$$\begin{aligned}\{\mathbf{21}^*\} =& \underbrace{(\mathbf{1},\mathbf{1},\mathbf{1},-2)}_{\mu_R}\oplus\underbrace{(\mathbf{3},\mathbf{1},\mathbf{1},4/3)}_{c_R}\oplus\underbrace{(\mathbf{3},\mathbf{2},\mathbf{1},1/3)}_{\begin{pmatrix}u\\ d\end{pmatrix}_L}\oplus\underbrace{(\mathbf{1},\mathbf{1},\mathbf{2},-1)}_{\begin{pmatrix}N_E\\ E\end{pmatrix}_R}\oplus\\ &\underbrace{(\mathbf{1},\mathbf{1},\mathbf{2},-1)}_{\begin{pmatrix}N_M\\ M\end{pmatrix}_R}\oplus\underbrace{(\mathbf{3},\mathbf{1},\mathbf{1},-2/3)}_{D_L}\oplus\underbrace{(\mathbf{3},\mathbf{1},\mathbf{1},-2/3)}_{S_L}\oplus\underbrace{(\mathbf{1},\mathbf{1},\mathbf{1},0)}_{N_{ML}},\end{aligned} \tag{3}$$

$$\{\mathbf{7}\} = \underbrace{(\mathbf{1},\mathbf{2},\mathbf{1},-1)}_{\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L}\oplus\underbrace{(\mathbf{3},\mathbf{1},\mathbf{1},-\,2/3)}_{d_R}\oplus\underbrace{(\mathbf{1},\mathbf{1},\mathbf{1},0)}_{N_{EL}}\oplus\underbrace{(\mathbf{1},\mathbf{1},\mathbf{1},0)}_{N_{ML}}. \tag{4}$$

$$\{\mathbf{1}\}=\underbrace{(\mathbf{1},\mathbf{1},\mathbf{1},0)}_{\nu_{eR}}\ . \tag{5}$$

From the product $\{\mathbf{63}\}\otimes\{\mathbf{63}\}=\{\mathbf{1}\}_s\oplus\{\mathbf{63}\}_s\oplus\{\mathbf{63}\}_a\oplus ...$, where the index indicate symmetric (s) or antisymmetric (a), we obtain the Higgs representations producing the mass terms for the fermions in the spinorial multiplet $\{\mathbf{63}\} = \{\mathbf{7}\}\oplus\{\mathbf{21}^*\}\oplus\{\mathbf{35}\}$ of $SU(7)$. With the help of the branching rules (1) - (5), we take

$$\chi_L\sim\{\mathbf{7}^*\}\supset(\mathbf{1},\mathbf{2},\mathbf{1},1),\quad \chi_R\sim\{\mathbf{21}\}\supset(\mathbf{1},\mathbf{1},\mathbf{2},1), \tag{6}$$

$$S_D\sim\{\mathbf{21}\}\supset(\mathbf{1},\mathbf{1},\mathbf{1},0),\quad S_M\sim\{\mathbf{1}\}\sim(\mathbf{1},\mathbf{1},\mathbf{1},0). \tag{7}$$

Finally we can have the following breaking chain with two singlets and two Higgs doublets :

$$SU(7) \underrightarrow{S_M} SU(5)\otimes SU(2)_R \otimes \mathcal{D} \underrightarrow{S_D} G_{SM}\otimes SU(2)_R$$
$$\underrightarrow{\chi_R} SU(3)_C\otimes SU(2)_L\otimes U(1)_Y \underrightarrow{\chi_L} SU(3)_C\otimes U(1)_{e.m} \ . \tag{8}$$

A fundamental problem in left-right models is to satisfy the condition $v_R >> v_L$. This can be done by introducing the concept of $\mathcal{D}$-parity [14, 28] The component of $\phi^{\alpha\beta} = \{\mathbf{21}\}$, $(\alpha,\beta = 1-7)$ that breaks $\mathcal{D}$-parity is given by $S_D = \phi^{67}$ which is odd under $\mathcal{D}$-parity [23] and S_M is a $SU(7)$ singlet that conserves $\mathcal{D}$-parity.

We can write an $SU(7)$ invariant Higgs potential which incorporates the $\mathcal{D}$-parity effect as

$$\mathcal{L} = \mu^2\{\mathbf{7}\}\times\{\mathbf{7}^*\} + \lambda_\chi(\{\mathbf{7}\}\times\{\mathbf{7}^*\})^2 + m_D^2\{\mathbf{21}\}^2 +$$
$$\eta_D\{\mathbf{21}\}^3 + \lambda_D\{\mathbf{21}\}^4 + m_M^2\{\mathbf{1}\}^2 + \eta_M\{\mathbf{1}\}^3 + \lambda_M\{\mathbf{1}\}^4 + M_D\{\mathbf{21}\}(\{\mathbf{7}\}\times\{\mathbf{7}^*\}) +$$
$$M_M\{\mathbf{1}\}(\{\mathbf{7}\}\times\{\mathbf{7}^*\}) + \lambda(\{\mathbf{1}\}\times\{\mathbf{21}\})(\{\mathbf{7}\}\times\{\mathbf{7}^*\}) +$$
$$(\varepsilon_D\{\mathbf{21}\}^2 + \varepsilon_M\{\mathbf{1}\}^2)(\{\mathbf{7}\}\times\{\mathbf{7}^*\}) + \varkappa[\{\mathbf{7}\}^4 + \{\mathbf{7}^*\}^4]. \tag{9}$$

Let us note that the term $\lambda(\{\mathbf{1}\}\times\{\mathbf{21}\})(\{\mathbf{7}\}\times\{\mathbf{7}^*\})$ is possible if the interactions of $\{\mathbf{1}\}\times\{\mathbf{21}\}$ and $\{\mathbf{7}\}\times\{\mathbf{7}^*\}$ are mediated by a gauge boson in the $\{\mathbf{21}\}$ representation of $SU(7)$.

3 Couplings in the Higgs Sector and g-2

3.1 The Higgs potential of a L-R model

There are two ways of breaking parity spontaneously: the first is to identify the discrete symmetry Z_2 that interchanges the groups $SU(2)_L$ and $SU(2)_R$ of the Lorentz group $O(3,1)$ as the parity operator $\mathcal{P}$, which allows the parity symmetry of the Higgs bosons to be $\chi_L \overset{\mathcal{P}}{\longleftrightarrow} \chi_R$ and also $W_L \overset{\mathcal{P}}{\longleftrightarrow} W_R$. Thus, when $SU(2)_R$ is broken in the symmetric L-R model, parity is also broken. The second possibility for a spontaneously breaking of the parity symmetry is through the v.e.v. of an odd scalar field that conserves the L-R symmetry. In this case it is not possible to have $\chi_L \overset{\mathcal{P}}{\longleftrightarrow} \chi_R$ if in the model there are complex Yukawa couplings. This type of parity is called $\mathcal{D}$-parity which is a generator of groups that contain the product $SU(2)_L\otimes SU(2)_R$ as a subgroup. This second possibility is very important because allow $\langle\chi_L\rangle \ll \langle\chi_R\rangle$ with different coupling constants for $SU(2)_L$ and $SU(2)_R$ and different masses for these Higgs fields.

Our model for the scalar potential includes two Higgs doublets and two Higgs singlets. These singlets and doublets transforms under $\mathcal{D}$-parity as $S_M \overset{\mathcal{D}}{\longleftrightarrow} S_M$, $S_D \overset{\mathcal{D}}{\longleftrightarrow} -S_D$ and $\chi_L \overset{\mathcal{D}}{\longleftrightarrow} \chi_R$, if in the model there is no CP violation or complex Yukawa couplings. In this case, $\mathcal{P}$ and $\mathcal{D}$-parity can be indistinctly considered . Let us suppose the following invariant potential under $G_{3221} = SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ for the Higgs fields

$$
\begin{aligned}
V(\chi_L, \chi_R, S_D, S_M) = \mu^2(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) - \lambda_\chi(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R)^2 - m_D^2 S_D^2 - \\
\eta_D S_D^3 - \lambda_D S_D^4 - m_M^2 S_M^2 - \eta_M S_M^3 - \lambda_M S_M^4 + M_D S_D(\chi_R^\dagger \chi_R - \chi_L^\dagger \chi_L) + \\
M_M S_M(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) + \lambda S_D S_M(\chi_R^\dagger \chi_R - \chi_L^\dagger \chi_L) + \\
(\varepsilon_D S_D^2 + \varepsilon_M S_M^2)(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) - \varkappa((\chi_L^4)^\dagger + \chi_L^4 + (\chi_R^4)^\dagger + \chi_R^4). \qquad (10)
\end{aligned}
$$

Our motivation to write this potential is the fact that S_M and S_D are not necessarily into the same irreducible multiplet of Higgs fields. In consequence it is also possible a mixing between these fields. If this is the case, when $\langle S_M \rangle = s_M$ and $\langle S_D \rangle = s_D$ the potential responsible for the Higgs masses for the fields χ_L and χ_R is

$$
\begin{aligned}
V_{\mathbf{mass}}(\chi_L, \chi_R) = (\mu^2 + \varepsilon_D s_D^2 + \varepsilon_M s_M^2 + M_M s_M)(|\chi_L|^2 + |\chi_R|^2) + \\
(M_D s_D + \lambda s_D s_M)(|\chi_R|^2 - |\chi_L|^2), \qquad (11)
\end{aligned}
$$

from which we find the mass terms,

$$m_R^2 = \mu^2 + \varepsilon_D s_D^2 + \varepsilon_M s_M^2 + M_M s_M + M_D s_D + \lambda s_D s_M, \qquad (12)$$

$$m_L^2 = \mu^2 + \varepsilon_D s_D^2 + \varepsilon_M s_M^2 + M_M s_M - M_D s_D - \lambda s_D s_M. \qquad (13)$$

Now we impose the hierarchy condition in the previous equations such that $m_R^2 \ll s_D^2 \ll s_M^2$. In this limit we can now have $\langle \chi_L \rangle = \upsilon_L \sim m_L \sim 100GeV$ and, let us say; $\langle \chi_R \rangle = \upsilon_R \sim m_R \sim 10TeV \gg \upsilon_L$. It is necessary to indicate that υ_L breaks the electroweak symmetry and υ_R breaks the L-R symmetry close to the TeV scale. It also must be noted that if S_D and S_M are into the same multiplet, the mixing terms in the potential possibility will be absent.

Let us now suppose that there is no CP violation and that all v.e.v. are considered to be real: $\langle \chi_L \rangle = \begin{pmatrix} 0 \\ \upsilon_L \end{pmatrix}$, $\langle \chi_R \rangle = \begin{pmatrix} 0 \\ \upsilon_R \end{pmatrix}$. Then it is possible to show that the minimum conditions for the potential are given by

$$
\begin{aligned}
\frac{\partial V}{\partial \upsilon_L} = 2\upsilon_L[\mu^2 - 2\lambda_\chi(\upsilon_L^2 + \upsilon_R^2) - M_D s_D + M_M s_M - \lambda s_D s_M + \\
\varepsilon_D s_D^2 + \varepsilon_M s_M^2 - 4\varkappa \upsilon_L^2] = 0, \qquad (14) \\
\frac{\partial V}{\partial \upsilon_R} = 2\upsilon_R[\mu^2 - 2\lambda_\chi(\upsilon_L^2 + \upsilon_R^2) + M_D s_D + M_M s_M + \lambda s_D s_M + \\
\varepsilon_D s_D^2 + \varepsilon_M s_M^2 - 4\varkappa \upsilon_R^2] = 0, \qquad (15)
\end{aligned}
$$

From these equations we have

$$\upsilon_L \frac{\partial V}{\partial \upsilon_R} - \upsilon_R \frac{\partial V}{\partial \upsilon_L} = 4\upsilon_L \upsilon_R[M_D s_D + \lambda s_D s_M - 2\varkappa(\upsilon_R^2 - \upsilon_L^2)] = 0 \qquad (16)$$

As we require non trivial solutions so that $\upsilon_L \neq \upsilon_R \neq 0$, we obtain the desired hierarchy

$$\upsilon_R^2 - \upsilon_L^2 = \frac{s_D(M_D + \lambda s_M)}{2\varkappa}. \tag{17}$$

An important point to be noted in the previous equation is that the effect of the breaking due to the singlet S_M is sub-dominant with relation to S_D which breaks $\mathcal{D}$-parity when $\langle S_D \rangle = s_D$. Additionally, if $s_D = 0$ the $\mathcal{D}$-parity is conserved and also the L-R symmetry producing $\upsilon_R = \upsilon_L$ as expected. Thus, we have shown that in our potential there is a possibility to construct models producing an hierarchy between the breaking scale of $SU(2)_R$ and the electroweak scale simply by using two Higgs singlets to generate the minimum of the potential. The crucial point in this sense is the inclusion of the mixing term $\lambda S_D S_M(\chi_R^\dagger \chi_R - \chi_L^\dagger \chi_L)$ which is possible if S_M and S_D are into different irreducible representations. In the same way as in the previous term, also the term $M_D S_D(\chi_R^\dagger \chi_R - \chi_L^\dagger \chi_L)$ breaks the L-R symmetry . It is fundamental also to fine tune the parameters of the model at the radiative level to assure that υ_R do not destabilizes the υ_L value. Thus, from equations (12) - (15) we must have

$$m_L^2 - 2(\lambda_\chi + 2\varkappa)\upsilon_L^2 = 2\lambda_\chi \upsilon_R^2\,, \tag{18}$$

3.2 The lepton couplings and $g-2$

An interesting model with mirror fermions based in the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_Y$ with a minimal set of Higgs fields was elaborated in [24]. The Lagrangian is constructed using two Higgs doublets χ_L, χ_R that satisfy the parity transformation $\chi_L \overset{P}{\longleftrightarrow} \chi_R$, and two Higgs singlets, the first of which is coupled to Dirac terms - S_D - and the other that couples to Majorana terms - S_M. This is a general approach to activate the see-saw mechanism for neutrino masses. In other L-R models [25][26] with only two doublets the minimum for the vacuum appear at $\upsilon_L = 0$, which is phenomenologically useless. It has also been shown by using a variational method [27] that this vacuum is unstable and that υ_L could gain a small value, in comparison to υ_R, when Higgs fields are coupled to fermion fields. The other possible solution is the inclusion of bi-doublets, increasing the number of fundamental parameters in the scalar sector. In the present approach, with two Higgs doublets and two Higgs singlets, the vacuum is stable as shown recently[28].

The Lagrangian containing terms relevant for the anomalous magnetic moment of the electron (or muon) is given by

$$\mathcal{L}_{\mathbf{Fch}} = f(\overline{l_L}\chi_L e_R + \overline{l_R}\chi_R E_L) + f'\,\overline{e_R} E_L S_D + h.c. \tag{19}$$

The Higgs sector that breaks the symmetry is

$$\chi_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \upsilon_L + X_L \end{pmatrix}, \qquad \chi_R = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \upsilon_R + X_R \end{pmatrix},$$
$$S_D = \frac{1}{\sqrt{2}} (s_D + X_D)\,, \tag{20}$$

where υ_L, υ_R and s_D are the vacuum expectation values and X_L, X_R and X_D are the respective neutral Higgs fields. The fermion mass terms are

$$\begin{pmatrix} \overline{e_L} & \overline{E_L} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} f\upsilon_L & 0 \\ f' s_D & f\upsilon_R \end{pmatrix} \begin{pmatrix} e_R \\ E_R \end{pmatrix} + h.c. \tag{21}$$

A rotation between the fermion fields will diagonalize the mass matrix

$$e_L = c_L e^0_L + s_L E^0_L, \quad e_R = c_R e^0_R - s_R E^0_R\,,$$
$$E_L = -s_L e^0_L + c_L E^0_L, \quad E_R = s_R e^0_R + c_R E^0_R\,, \tag{22}$$

where $s_{L,R} = \sin\theta_{L,R}\,,\ c_{L,R} = \cos\theta_{L,R}$. The weak eigenstates are $e_{L,R}$ and $E_{L,R}$ while $e^0_{L,R}$ and $E^0_{L,R}$ are the mass eigenstates. Then, the terms of the previous Lagrangian contributing to the magnetic moment and CP conserving are

$$\mathcal{L}_{\mathbf{eE}} = \frac{f}{\sqrt{2}} [(s_L c_R \overline{E^0_L} e^0_R - c_L s_R \overline{e^0_L} E^0_R) X_L + (s_R c_L \overline{e^0_R} E^0_L - c_R s_L \overline{E^0_R} e^0_L) X_R]$$
$$+ \frac{f'}{\sqrt{2}} (c_L c_R \overline{e^0_R} E^0_L + s_L s_R \overline{E^0_R} e^0_L) X_D + h.c. \tag{23}$$

The Feynman diagram producing a new contribution to the anomalous magnetic moment of leptons consistent with the requirement of a well defined Higgs potential is shown in Fig.1.

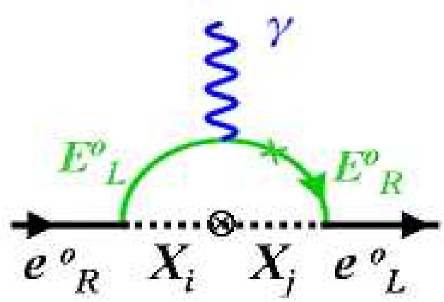

Fig. 1 Generic Feynman diagram contributing to the lepton anomaly. The Higgs fields are in the mass eigenstates basis. The index is $i \neq j = L, R, D$.

Note the mixing term between Higgs fields that arise from the potential (10). This mixing is crucial to give a finite radiative mass to leptons (the same diagram as in Fig.1, but without the photon line). Figure 1 is the relevant contribution for the anomalous magnetic moment in the hierarchy $\upsilon_D \gg \upsilon_R \gg \upsilon_L$ with $M_D \gg M_R \gg M_L$. This contribution for the electron case is, with the additional condition $m_E \gg m_e$ for the mirror partner of the electron E, is given by [29][30]

$$\Delta a_e = \frac{\xi_e}{16\pi^2}\frac{m_e}{m_E} \times \frac{\left(1-z_e^2\right)^2 - 2z_e^2\left(1-z_e^2\right) - 2z_e^4\ln(z_e^2)}{2\left(1-z_e^2\right)^3}, \tag{24}$$

where $z_e = \frac{M_L}{m_E}$ and the parameter ξ_e is function of $f, \theta_{L,R}$ and of the mixing angle between X_L and X_R that can be easily obtained from the Lagrangian $\mathcal{L}_{\mathbf{eE}}$. The corresponding contribution to the electron radiative mass is given by [30][13]

$$m_e^{\textbf{1-loop}} \simeq \frac{\xi_e}{16\pi^2} m_E \left[\frac{M_L^2}{m_E^2 - M_L^2}\ln\left(\frac{m_E^2}{M_L^2}\right) - \frac{M_R^2}{m_E^2 - M_R^2}\ln\left(\frac{m_E^2}{M_R^2}\right)\right]. \tag{25}$$

We will use the very small contribution to $m_e^{\textbf{1-loop}}$ and to the anomalous magnetic moment to obtain constrains over ξ_e, M_L, M_R and m_E. We obtain analogous expressions for the muon.

4 Bounds from $g-2$ and radiative masses.

In this section we use two simple arguments in order to obtain bounds over the parameters in our model. The first one is the value of the leptons anomaly and the second is the small value for the radiative mass. Let us take $-10^{-11} \lesssim \Delta a_e \lesssim 3\times 10^{-11}$ for the electron anomaly. In Fig. 2 we show the possible range of values for ξ_e as a function of M_L/m_E for two different cases: $m_E = 100GeV$ and $200GeV$. In this paper we consider the mass parameter that fixes the Standard Model Higgs boson to be in the range $47GeV \leqslant M_L \leqslant 200GeV$ [31].

We have also obtained constraints for ξ_e from the radiative mass given in equation(25) as is shown in Fig.3. A small radiative mass for the electron is possible in our model. For example, by taking $m_e^{\textbf{1-loop}} = 0.05MeV$, $m_E = 100GeV$ with $47GeV \leqslant M_L \leqslant 200GeV$ we found $-2\times10^{-5} \lesssim \xi_e \lesssim -4.5\times10^{-5}$ in the range of values $6 \leqslant \frac{M_R}{m_E} \lesssim 10$. This is showed in Fig.3a. For the case $m_E = 200GeV$ with the same electron radiative mass we have the results of Fig.3b. In both cases, the range of values of ξ_e is compatible with the values coming from the electron anomaly, as is showed in Fig. 2a and Fig.2b. Thus, a Higgs SM with mass in the indicated range and its mirror partner with a mass $600GeV \leqslant M_R \leqslant 1TeV$ can account for the electron anomaly and give a small electron radiative mass.

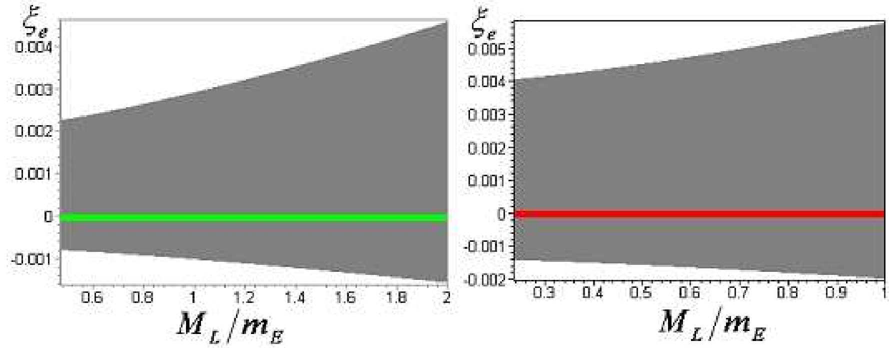

Fig. 2 Allowed values for ξ_e coming from the electron $g_e - 2$ as a function of M_L/m_E for two cases: a) $m_E = 100\,GeV$ and b) $m_E = 200\,GeV$.

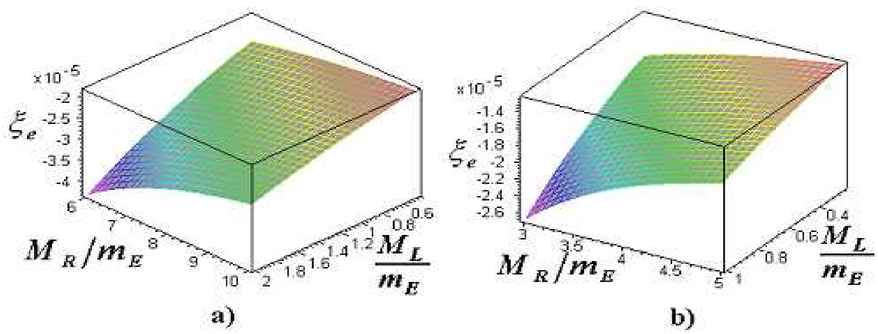

Fig. 3 Range of allowed values for ξ_e from the radiative mass contribution, taking $m_e^{\textbf{1-loop}} = 0.05MeV$, as a function of M_R/m_E and M_L/m_E for: a) $m_E = 100GeV$ and b) $m_E = 200GeV$.

For the muon case let us assume the value $-2 \times 10^{-9} \lesssim \Delta a_\mu \lesssim 6 \times 10^{-9}$ for the anomalous magnetic moment. The range of values for ξ_μ coming from muon anomaly is shown in Fig.4. For the radiative muon mass we have the results shown in Fig. 5.

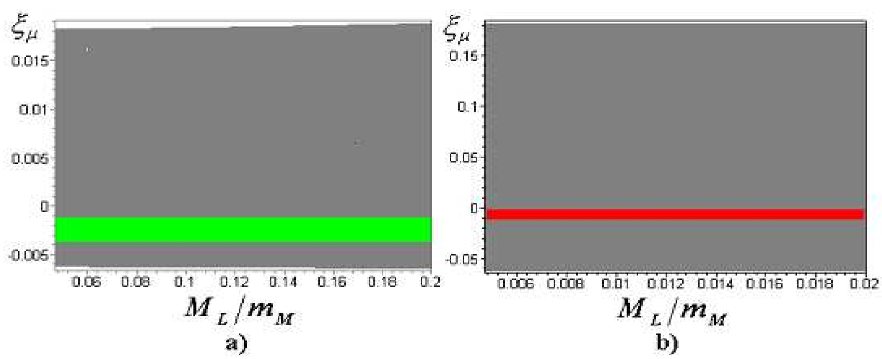

Fig. 4 Range of values for ξ_μ coming from $g_\mu - 2$ as a function of M_L/m_M, for: a) $m_M = 1TeV$ and b) $m_M = 10TeV$.

The shadowed area in Fig. 4a gives a range of values $-1.5\times10^{-3}\leqslant\xi_\mu\leqslant-3.5\times10^{-3}$ coming from the muon radiative mass for $M_R=600GeV-1TeV$, $47GeV\leqslant M_L\leqslant200GeV$ and a small radiative mass $m_\mu^{\textbf{1-loop}}=10MeV$. This is compatible with the values for ξ_μ coming from of muon anomaly.

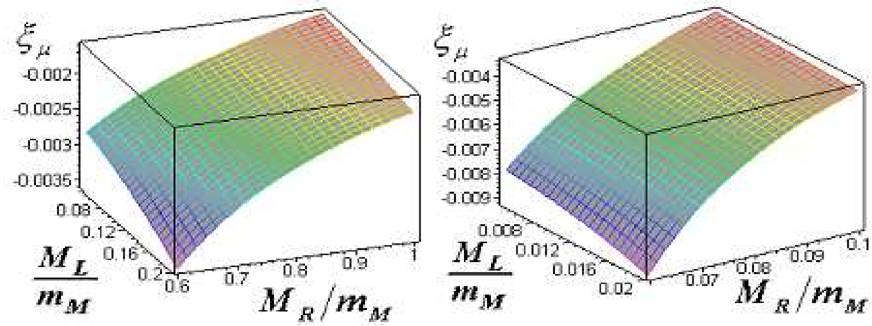

Fig. 5 Range of values of ξ_μ from the muon radiative mass, as a function of M_R/m_M and M_L/m_M. We consider $m_\mu^{\textbf{1-loop}}=10MeV$ for: a) $m_M=1TeV$ and b) $m_M=10TeV$.

Let us notice from Fig.5 that it is possible to obtain smaller values for the muon radiative mass $1\leqslant m_\mu^{\textbf{1-loop}}\leqslant10MeV$ by taking $m_M=1TeV-10TeV$ for a Standard Model Higgs with mass between $47-200GeV$ and M_R between $600GeV-1TeV$. These cases are totally compatible with the present value of the muon anomaly.

5 Comments on new corrections

Left-right symmetric models with mirror fermions will have other contributions to the muon anomaly.

There are corrections from vacuum polarization loops with E and $M-$mirror, similar to the QED contribution. The insertion of $E-$mirror vacuum polarization loop into the muon vertex correction, give $\Delta a_{\mathbf{v.p.l}}\simeq[\frac{1}{45}\left(\frac{m_\mu}{m_E}\right)^2+\frac{1}{70}\frac{m_\mu^4}{m_E^4}\times\ln\frac{m_E}{m_\mu}]\left(\frac{\alpha}{\pi}\right)^2$. For the range of parameters of the new mirror sector considered in this paper we have the value $\sim10^{-14}-10^{-13}$.

New gauge bosons will also contribute to the muon anomaly with diagrams analogous to the standard model gauge bosons. As the new gauge bosons must have masses higher than the standard gauge bosons we will have again very small contributions.

The new hadronic contribution from mirror matter will suffer the same limitations as in the standard model. One can not calculate the hadronic contribution directly from QCD due to the non-perturbative region of parameters. The standard procedure is to use dispersion relations and experimental data from e^+e^- collisions or the pion spectral functions. Hadronic vacuum polarization loops with new mirror quarks could contribute to the muon anomaly. But we would need to know the e^+e^- annihilation into mirror quark-antiquark and their consequent hadronization. Similar possibilities could occur for the hadronic $\tau-$ decay. We expect that this new hadronic contributions from mirror quarks will be very small in virtue of the hierarchy masses present in our model, in comparison with the ordinary quarks masses.

6 Conclusions

In conclusion, we found that in a $SU(7)$ grand unified symmetric left-right model it is possible to have a Standard Model Higgs boson with mass between $47-200GeV$, a new mirror Higgs boson with mass between $600GeV-1TeV$; a mirror electron with $m_E \simeq 100GeV$ and a mirror muon with mass $m_M \simeq 1TeV$. These values are compatible with the electron and muon anomaly and give a small radiative mass contribution. These possible new high mass states can be searched in the LHC energies [32]and new high energy lepton colliders[33]

As was showed in [10], the important linear relation between masses of ordinary and mirror fermions for the electron or muon anomaly is related to the breaking of the Weinberg symmetry [34], if the radiative correction to the masses of the ordinary fermions are small, as it is the case in the present model.

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References

[1] T. Kinoshita and M. Nio. Phys. Rev. D **73**(2006)013003, T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Nucl. Phys. B **740**(2006)138.

[2] T. Gribook and A. Czarnecki, Phys. Rev. D **72**(2005)053016

[3] B. Lee Roberts (for the Muon (g-2) Collaborations), hep-ex/0501012; G. W. Bennet et al., (BNL Muon g-2 Collaboration), Phys.Rev.Lett.92 (2004) 161802; E. Sichtermann et al., (BNL Muon (g-2) Collaboration), hep-ex/0309008, G. W. Bennet et al., Phys. Rev. Lett. 89 (2002) 129903.

[4] J. F. de Troconiz anf F. J. Yndurain, Phys.Rev. D **71** (2005) 073008, M. Knecht, Lect. Notes Phys. **629**(2004)37, V. Barger, Ch. Kao, P. Langacker and Hye-Sung Lee, Phys.Lett. B **614** (2005) 67, A. Czarnecki, Nuc.Phys. B (Proc. Suppl.) **144** (2005) 201.

[5] A. Nyffeler, Acta Phys. Polon. B **34** (2003) 5197.

[6] K. Melnikov and A. Vainshtein, Phys. Rev. D 70(2004)113006.

[7] M. Davier, S. Eidelman, A. Höcker and Z. Zhang, Euro. Phys. J. C. **31** (2003) 503, T. Teubner, Eur Phys. J. C **33** (2004) 653, A. Höcker,

hep-ph/0410081, K. Hagiwara, A.D. Martin, Daisuke Nomura and T. Teubner, arXiv:hep-ph/0312250 v3

[8] M. Passera, hep-ph/0509372 v2, , M. Passera, Jour. Phys. G.: Nucl. Part. Phys. **31** (2005) R75.

[9] Thomas Rizzo, Phys. Rev. D **33**(1986)3329, I. Vendramin, IL Nuovo Cimento, **100**(1988)757

[10] Helder Chavez, Cristine N. Ferreira and José A. Helayel-Neto, Phys. Rev. D **74**(2006)033006-1, hep-ph/0410373 v2.

[11] J. L. Lopez, D. V. Nanopoulos and X. Wang, Phys. Rev. D **49**(1994)366, J. Ellis, J. Hagellin and D. V. Nanopoulos, Phys. Lett. B **116**(1982)283, G.-C. Cho, K. Hagiwara and M. Hayakawa, Phys. Lett. B **478**(2000)231, T. Ibrahim and P. Nath, Phys. Rev. D **62**(2000)015004, U. Chattopadhyay, D. K. Ghosh and S. Roy, Phys. Rev. D **62**(2000)115001, Tai-Fu Feng, Xue-Qian Li, Lin Lin, Jukka Maalampi, He-Shan Song, hep-ph/0604171.

[12] J. Maalampi, J. T. Peltoniemi and M. Ross, Phys. Lett. B **220**(1989)441, F. Czikor and Z. Fodor, Phys. Lett. B **287**(1992)358.

[13] Andrzej Czarnecki and William Marciano, Phys. Rev. D **64**(2001)013014.

[14] D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. **52**(1984)1072, D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. D **30**(1984)1052, D. Chang, R. N. Mohapatra, J. M. Gibson, R. E. Marshak and M. K. Parida, Phys. Rev. D **31**(1985)1718, D. Chang and A. Kumar, Phys. Rev. D **33**(1986)2695.

[15] K. Enqvist and J. Maalampi, Nucl. Phys. B **191**(1981)189, J. Maalampi and M. Roos, Phys. Rept. **186**(1990)53 .

[16] J. Chakrabarti, M. Popovic and R. N. Mohapatra, Phys. Rev. D **21**(1980)3212, I. Umemura and K. Yamamoto, Phys. Lett. B **100**(1981)34

[17] I. Bars and Günaydin, Phys. Rev. Lett. **45** (1980)859, S. M. Barr, Phys. Rev. D **37**(1988)204.

[18] M. Collie and R. Foot, Phys. Lett. B **432**(1998)134.

[19] R. Slansky, Phys. Rep. **79** (1981) 1.

[20] N. S. Baaklini, Phys. Rev. D **21**(1980)1932, A. Umemura and K. Yamamoto, Progress of Theor. Phys. **66**(1981)1430.

[21] K. Yamamoto, Mirror Fermions in the SU(7) GUT and Their Effects on Flavour Changing Process, Kyoto University, unpublished, 1983.

[22] Kyuwan Hwang and Jihn E. Kim, Phys. Lett. B **540** (2002) 289, M. Claudson, A. Yildiz and P. H. Cox, Phys. Lett. B **97(**1980)224.

[23] Jihn E. Kim, Phys. Rev. D **26** (1982)2009.

[24] Y. A. Coutinho, J. A. Martins Simoes and C. M. Porto, Eur. Phys. J. C **18**(2001)779; F. M. L. Almeida, Y. A. Coutinho, J. A. Martins Simoes, J. Ponciano, A. J. Ramalho, S. Wulck and M. A. B. Vale, Eur. Phys. J. C **38**(2004)115; J. A. Martins Simoes and J. Ponciano, Eur. Phys. J. C **32S1**(2004)91.

[25] F. Siringo, Phys. Rev. Lett. **92** (2004) 119101.

[26] B. Brahmachari, E. Ma and U. Sarkar, Phys. Rev. Lett. **91**(2003)011801 .

[27] F. Siringo and L. Marotta, hep-ph/0605276.

[28] H. Chávez and J. A. Martins Simões hep-ph/0606112 v2.

[29] Jacques P. Leveille, Nuc. Phys. B **137**(1978)63

[30] T. W. Kephart and H. Päs, Phys. Rev. D **65**(2002)093014.

[31] Jens Erler, hep-ph/0607332 v2.

[32] Y. A. Coutinho, J. A. Martins Simoes, C. M. Porto and P. P. Queiroz Filho, Phys. Rev. D **57**, 6975 (1998).

[33] F. M. L. Almeida, Y. A. Coutinho, J. A. Martins Simoes, S. Wulck and M. A. B. do Vale, Eur. Phys. J. C **30**, 327 (2003)

[34] S. Weinberg, The Quantum Theory of Fields (Cambridge University Press, Cambridge, England, 1995, Vol.1, p. 520).